EXERCISE – I　SINGLE CORRECT (OBJECTIVE QUESTIONS)

1. $f(x) = 1 + 2x^2 + 4x^4 + 6x^6 + \ldots\ldots\ldots\ldots\ldots + 100x^{100}$ is polynomial in a real variable x, then f(x) has
(A) neither a maximum nor a minimum
(B) only one maximum
(C) only one minimum
(D) one maximum and one minimum

2. On the interval [0, 1] the function $x^{25}(1 - x)^{75}$ takes its maximum value at
(A) 0　(B) 1/2　(C) 1　(D) 1/4

3. The product of minimum value of x^x and maximum value of $\left(\frac{1}{x}\right)^x$ is
(A) e　(B) e^{-1}　(C) 1　(D) e^2

4. The minimum value of the function defined by f(x) = max (x, x + 1, 2 – x) is
(A) 0　(B) 1/2　(C) 1　(D) 3/2

5. Let $f(x) = \begin{cases} \sin\frac{\pi}{2}x & , \quad 0 \le x < 1 \\ 3 - 2x & , \quad x \ge 1 \end{cases}$ then
(A) f(x) has local maxima at x = 1
(B) f(x) has local minima at x = 1
(C) f(x) does not have any local extrema at x = 1
(D) f(x) has a global minima at x = 1

6. The greatest and the least values of the function, $f(x) = 2 - \sqrt{1 + 2x + x^2}$, $x \in [-2, 1]$ are
(A) 2, 1　(B) 2, –1　(C) 2, 0　(D) None of these

7. Let f(x) = {x}, For f(x), x = 5 is
(where {∗} denotes the fractional part)
(A) a point of local maxima
(B) a point of local minima
(C) neither a point of local minima nor maxima
(D) None of these

8. The critical points of $f(x) = \frac{|x - 1|}{x^2}$ lies at
(A) $x \in \{1, 2\}$　(B) $x \in \{0, 1\}$
(C) $x \in \{2\}$　(D) None of these

9. The difference between the greatest and least values of the function f(x) = sin 2x – x on [–π/2, π/2] is
(A) $\frac{\sqrt{3} + \sqrt{2}}{2}$　(B) $\frac{\sqrt{3} + \sqrt{2}}{2} + \frac{\pi}{6}$
(C) $\frac{\pi}{2}$　(D) π

10. The radius of a right circular cylinder of greatest curved surface which can be inscribed in a given right circular cone is
(A) one third that of the cone
(B) $1/\sqrt{2}$ times that of the cone
(C) 2/3 that of the cone　(D) 1/2 that of the cone

11. The dimensions of the rectangle of maximum area that can be inscribed in the ellipse $(x/4)^2 + (y/3)^2 = 1$ are
(A) $\sqrt{8}, \sqrt{2}$　(B) 4, 3　(C) $2\sqrt{8}, 3\sqrt{2}$　(D) $\sqrt{2}, \sqrt{6}$

12. The largest area of a rectangle which has one side on the x–axis and the two vertices on the curve $y = e^{-x^2}$ is
(A) $\sqrt{2}\ e^{-1/2}$　(B) $2\ e^{-1/2}$
(C) $e^{-1/2}$　(D) None of these

13. The co–ordinates of the point on the curve $x^2 = 4y$, which is at least distance from the line y = x – 4 is
(A) (2, 1)　(B) (–2, 1)　(C) (1, –2)　(D) (1, 2)

14. $f(x) = \begin{cases} \tan^{-1} x & , \quad |x| < \frac{\pi}{2} \\ \frac{\pi}{2} - |x| & , \quad |x| \ge \frac{\pi}{2} \end{cases}$ then
(A) f(x) has no point of local maxima
(B) f(x) has only one point of local maxima
(C) f(x) has exactly two points of local maxima
(D) f(x) has exactly two points of local minima

15. Let f(x) = $\begin{cases} x^3 - x^2 + 10x - 5 & , \quad x \le 1 \\ -2x + \log_2(b^2 - 2) & , \quad x > 1 \end{cases}$ the set of values of b for which f(x) has greatest value at x = 1 is given by

(A) $1 \le b \le 2$ (B) b = {1, 2}

(C) $b \in (-\infty, -1)$ (D) $[-\sqrt{130}, -\sqrt{2}) \cup (\sqrt{2}, \sqrt{130}]$

16. The set of values of p for which the extrema of the function, $f(x) = x^3 - 3px^2 + 3(p^2 - 1)x + 1$ lie in the interval (–2, 4) is

(A) (–3, 5) (B) (–3, 3) (C) (–1, 3) (D) (–1, 4)

17. Four points A, B, C, D lie in that order on the parabola $y = ax^2 + bx + c$. The co–ordinates of A, B & D are known as A(–2, 3); B(–1, 1) and D(2, 7). The co–ordinates of C for which the area of the quadrilateral ABCD is greatest is

(A) (1/2, 7/4) (B) (1/2, –7/4)

(C) (–1/2, 7/4) (D) None of these

18. In a regular triangular prism the distance from the centre of one base to one of the vertices of the other base is ℓ. The altitude of the prism for which the volume is greatest is

(A) $\frac{\ell}{2}$ (B) $\frac{\ell}{\sqrt{3}}$ (C) $\frac{\ell}{3}$ (D) $\frac{\ell}{4}$

19. Two vertices of a rectangle are on the positive x–axis. The other two vertices lie on the lines y = 4x and y = –5x + 6. Then the maximum area of the rectangle is

(A) 4/3 (B) 3/5 (C) 4/5 (D) 3/4

20. A variable point P is chosen on the straight line x + y = 4 and tangents PA and PB are drawn from it to circle $x^2 + y^2 = 1$. Then the position of P for the smallest length of chord of contact AB is

(A) (3, 1) (B) (0, 4) (C) (2, 2) (D) (4, 0)

21. The lower corner of a leaf in a book is folded over so as to just reach the inner edge of the page. The fraction of width folded over if the area of the folded part is minimum is

(A) 5/8 (B) 2/3 (C) 3/4 (D) 4/5

22. If x_1 and x_2 are abscissa of two points on the curve $f(x) = x - x^2$ in the interval [0, 1], then maximum value of the expression $(x_1 + x_2) - (x_1^2 + x_2^2)$ is

(A) 1/2 (B) 1/4 (C) 1 (D) 2

23. The maximum area of the rectangle whose sides pass through the angular points of a given rectangle of sides a and b is

(A) 2 (ab) (B) $\frac{1}{2}(a + b)^2$

(C) $\frac{1}{2}(a^2 + b^2)$ (D) None of these

24. Least value of the function, $f(x) = 2^{x^2} - 1 + \frac{2}{2^{x^2} + 1}$ is

(A) 0 (B) 3/2 (C) 2/3 (D) 1

25. If p and q are positive real numbers such that $p^2 + q^2 = 1$, then the maximum value of (p + q) is

(A) 2 (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}$

26. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minimum at

(A) x = –2 (B) x = 0 (C) x = 1 (D) x = 2

27. If x is real, the maximum value of $\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$ is

(A) 41 (B) 1 (C) 17/7 (D) 1/4

28. A triangular park is enclosed on two sides by a fence and on the third side by a straight river bank. The two sides having fence are of same length x. The maximum area enclosed by the park is

(A) $\sqrt{\frac{x^3}{8}}$ (B) $\frac{1}{2}x^2$ (C) πx^2 (D) $\frac{3}{2}x^2$

29. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where a > 0, attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a equals

(A) 3 (B) 1 (C) 2 (D) 1/2

30. The maximum value $x^3 - 3x$ in the interval [0, 2] is

(A) 1 (B) 2 (C) 0 (D) –2

31. Minimum value of $\dfrac{1}{3\sin\theta - 4\cos\theta + 7}$ is

(A) $\dfrac{7}{12}$ (B) $\dfrac{5}{12}$ (C) $\dfrac{1}{12}$ (D) $\dfrac{1}{6}$

32. The minimum value of $(x - p)^2 + (x - q)^2 + (x - r)^2$ will be at x equals to

(A) pqr (B) $\sqrt[3]{pqr}$ (C) $\dfrac{p+q+r}{3}$ (D) $p^2+q^2+r^2$

33. The number of values of x where

$f(x) = \cos x + \cos \sqrt{2}\, x$ attains its maximum value is

(A) 1 (B) 0 (C) 2 (D) infinite

34. The co-ordinate of the point for minimum value of $z = 7x - 8y$ subject to the conditions $x + y - 20 \le 0$, $y \ge 5$, $x \ge 0$, $y \ge 0$

(A) (20, 0) (B) (15, 5) (C) (0, 5) (D) (0, 20)

35. The maximum value of

$\cos \alpha_1 . \cos \alpha_2 . \cos \alpha_3 .... \cos \alpha_n$ under the restriction

$0 \le \alpha_1, \alpha_2, ... \alpha_n \le \dfrac{\pi}{2}$ and $\cot \alpha_1 \cot \alpha_2 .... \cot \alpha_n = 1$ is

(A) $1/2^{n/2}$ (B) $1/2^n$ (C) $-1/2^n$ (D) 1

36. The point on the curve $4x^2 + a^2 y^2 = 4a^2$, $4 < a^2 < 8$, that is farthest from the point (0, –2) is

(A) (2, 0) (B) (0, 2) (C) (2, – 2) (D) (–2, 2)

37. The equation $x^3 - 3x + [a] = 0$, will have three real and distinct roots if

(where [∗] denotes the greatest integer function)

(A) $a \in (-\infty, 2)$ (B) $a \in (0, 2)$

(C) $a \in (-\infty, 2) \cup (0, \infty)$ (D) $a \in [-1, 2)$

38. Let $f(x) = \sin\dfrac{\{x\}}{a} + \cos\dfrac{\{x\}}{a}$. Then the set of values of a for which f can attain its maximum values is (where a>0 and { ∗ } denotes the fractional part function)

(A) $\left(0, \dfrac{4}{\pi}\right)$ (B) $\left(\dfrac{4}{\pi}, \infty\right)$ (C) $(0, \infty)$ (D) None of these

39. A possible ordered pair (a, b) such that all the local extremum values of the function

$f(x) = x^3 + ax^2 - 9x + b$ are positive and the local minimum value occurs at point x = 1 is

(A) (3, 5) (B) (3, 6) (C) (3, 4) (D) (3, 3)

40. A function is defined as $f(x) = ax^2 - b|x|$ where a and b are constants then at x = 0 we will have a maxima of f(x) if

(A) a > 0, b > 0 (B) a > 0, b < 0

(C) a < 0, b < 0 (D) a < 0, b > 0

41. A and B are the points (2, 0) and (0, 2) respectively. The coordinates of the point P on the line $2x + 3y + 1 = 0$ are

(A) (7, –5) if |PA – PB| is maximum

(B) $\left(\dfrac{1}{5}, \dfrac{1}{5}\right)$ if |PA – PB| is maximum

(C) (7, –5) if |PA – PB| is minimum

(D) $\left(\dfrac{1}{5}, \dfrac{1}{5}\right)$ if |PA – PB| is minimum

42. The maximum value of $f(x) = 2bx^2 - x^4 - 3b$ is g(b), where b > 0, if b varies then the minimum value of g(b) is

(A) $\dfrac{3}{2}$ (B) $\dfrac{9}{2}$ (C) $-\dfrac{9}{4}$ (D) $-\dfrac{9}{2}$

43. Number of solution(s) satisfying the equation, $3x^2 - 2x^3 = \log_2 (x^2 + 1) - \log_2 x$ is

(A) 1 (B) 2 (C) 3 (D) None of these

44. If $a^2x^4 + b^2 y^4 = c^6$, then the maximum value of xy is

(A) $\dfrac{c^3}{2ab}$ (B) $\dfrac{c^3}{\sqrt{2|ab|}}$ (C) $\dfrac{c^3}{ab}$ (D) $\dfrac{c^3}{\sqrt{|ab|}}$

45. Maximum and minimum value of f(x) = max (sin t), $0 < t < x$, $0 \le x \le 2\pi$ are

(A) 1, 0 (B) 1, –1 (C) 0, –1 (D) None of these

46. The greatest value of $f(x) = (x + 1)^{1/3} - (x - 1)^{1/3}$ in [0, 1] is

(A) 1 (B) 2 (C) 3 (D) $2^{1/3}$

47. The function 'f' is defined by $f(x) = x^p (1 - x)^q$ for all $x \in R$, where p, q are positive integers, has a maximum value, for x equal to

(A) $\dfrac{pq}{p+q}$ (B) 1 (C) 0 (D) $\dfrac{p}{p+q}$

48. The maximum slope of the curve $y = -x^3 + 3x^2 + 2x - 27$ will be
(A) $-165/8$ (B) -27 (C) 5 (D) None of these

49. The least area of a circle circumscribing any right triangle of area S is
(A) πS (B) $2\pi S$ (C) $\sqrt{2}\pi S$ (D) $4\pi S$

50. Two points A(1, 4) & B(3, 0) are given on the ellipse $2x^2 + y^2 = 18$. The co-ordinates of a point C on the ellipse such that the area of the triangle ABC is greatest is
(A) $(\sqrt{6}, \sqrt{6})$ (B) $(-\sqrt{6}, \sqrt{6})$
(C) $(\sqrt{6}, -\sqrt{6})$ (D) $(-\sqrt{6}, -\sqrt{6})$

51. The lateral edge of a regular hexagonal pyramid is 1 cm. If the volume is maximum, then its height must be equal to
(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) 1

52. Let $f(x) = 5x - 2x^2 + 2$; $x \in N$ then the maximum value of f(x) is
(A) 8 (B) 5 (C) 4 (D) $\frac{41}{8}$

53. The maximum value of f(x), if $f(x) + f\left(\frac{1}{x}\right) = \frac{1}{x}$, $x \in$ domain of f
(A) -1 (B) 2 (C) 1 (D) 1/2